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INFILTRATION OF A SALT SOLUTION INTO
A SWELLING SOIL

V. I. Pen'kovskii

The physicochemical and hydraulic parameters of soils are largely dependent on the content of clay minerals, which make up more than half of all sediments in the earth's crust. The lattice surfaces of the clay particles bear negatively charged oxygen ions, and therefore soils are basically cation-exchange materials capable of taking up cations from an electrolyte and exchanging equivalent amounts of positively charged ions. A dry clay soil on wetting by water or a solution swells and absorbs the water and solutes. The cause of the swelling is hydration of the ions in the hydrophylic groups in the soil. The extent of the swelling is dependent on the hydrated radius of the ion and the exchange capacity of the soil-absorbing complex. The swelling is accompanied by coalescence of colloidal particles, which leads to an increase in the amount of relatively immobile water [1, 2] and a substantial reduction in the filtration capacity of the soil.

The converse phenomenon is peptization or particle dispersion, which is accompanied by a reduction in the amount of bound water and improvement in the permeability, as is observed when an electrolyte infiltrates into a soil containing fresh water. Experiments in the field and in the laboratory with soil systems [3, 4] show that the balance between peptization and coalescence may produce large reductions or increases in the permeability of a given specimen of natural soil. It is important to consider these phenomena in developing methods of calculating the water-salt conditions in soils during irrigation and draining, as well as in research on the stability of earth dams and related problems.

Here we consider a model case of the infiltration of fresh water into a clay soil whose skeleton retains a certain amount of salt solution with a given concentration.

1. Formulation. The theory of double electrical layers implies that the concentration C_i in moles/liter of ion i in the solution surrounding the negatively charged surface of a clay particle is [5] given by

$$C_i = C_i^0 \exp\left[-z_i e\left(\psi - \psi^0\right)/kT\right],$$

where z_i is the valency of the ion, e is the electronic charge, $\psi = \psi(y)$ is the electrical potential, k is Boltzmann's constant, T is absolute temperature, C_i^0 and ψ^0 are the values of the equilibrium concentration and electrical potential, respectively, as measured far from the surface, and y is a coordinate measured along the normal to the surface of the particle. The solution in an elementary volume at a certain distance y will be attracted to the charged surface under the action of the electric field with a force

$$dp = -\sum_i z_i C_i \frac{N_A e}{1000}\, dy\, \frac{d\psi}{dy} = -\frac{N_A e}{1000}\sum_i z_i C_i^0 \exp\left[-\frac{z_i e\left(\psi - \psi^0\right)}{kT}\right] d\psi,$$

where N_A is Avogadro's number and the summation is taken over each ion ($z_i > 0$ for a cation and $z_i < 0$ for an anion). After integration of this expression with respect to p from p^0 to p and with respect to ψ from ψ^0 to ψ we get the value of the excess pressure (swelling pressure) $\Delta p = p - p_0$ acting near the surface of a colloidal particle:

$$\Delta p = \frac{RT}{1000} \sum_i \left(C_i - C_i^0\right)_i \tag{1.1}$$

where $R = kN_A$ is the universal gas constant and p^0 is the pressure far from the surface. Therefore, if the overall concentration $N = \Sigma C_i$ of the solution adjoining the surface of the clay particle is greater than the overall concentration $C = \Sigma C_i^0$ of the solution in the transit pores of the soil we have $\Delta p > 0$, and the resulting additional pressure will increase the thickness of the solvate layer of liquid surrounding the particle, while the formation of relatively immobile solvate layers surrounding several particles simultaneously results in the coagulation. Conversely, in the case $N < C (\Delta p < 0)$ the solvate layers are destroyed and the particles are dispersed (colloid peptization).

Accordingly we assume that the volume amount of solution θ bound to the soil skeleton (in unit volume of physical space) may be represented to a first approximation as the linear function of the swelling pressure Δp. On the basis of (1.1) we write this assumption as

$$\theta = \theta^0 + \beta(N - C), \tag{1.2}$$

where θ^0 is the volume amount of solution bound to the soil under standard conditions, where the salt concentrations in the mobile and immobile solutions are identical, while β is an experimental constant dependent on the mechanical and mineral compositions of the soil, and also on the ionic composition of the solution and on temperature.

To determine the dependence of the filtration coefficient K on the amount of bound solution θ we use the Slichter–Coseni theoretical formula [6]

$$K = am_e^n, \tag{1.3}$$

where m_e is the effective porosity of the medium, a is a certain constant dependent on the characteristic diameter of the particles and on the packing, and n is a parameter numerically equal to 3.3 (according to Slichter) or 4.0 (according to Coseni). With an overall bulk water content w, the effective porosity is $m_e = w - \theta$. We assume that we know the value $K = K_0 = a(m - \theta^0)^n$ under the conditions of the standard experiment, where m is the total porosity of the medium. Then substitution of $a = K_0(m - \theta^0)^{-n}$ and m_e into (1.3) gives

$$K(w, \theta) = K_0 [(w - \theta)/(m - \theta^0)]^n, \tag{1.4}$$

which for $\theta = \theta^0$ $(\beta = 0)$ and n = 3.5 coincides with Aver'yanov's formula for the filtration coefficient in an incompletely saturated nonswelling soil [7].

The following equations [7] constitute the basic system describing the simultaneous water and salt transport in the porous medium: Fick's law for the mass flux j of the solute

$$j = -D(w, q)\partial C/\partial x + qC,$$

and the generalized form of Darcy's law for the bulk rate of infiltration q of the solution

$$q = -K(w, \theta)(\partial p/\partial x - 1)$$

(p is the pressure in the liquid, while the x axis is vertical downwards, and we neglect changes in the density and viscosity of the solution due to the concentration), together with the laws of conservation for the solute and the solvent (water) respectively:

$$-\frac{\partial j}{\partial x} = \frac{\partial}{\partial t}[(w - \theta)C + \theta N], \quad -\frac{\partial q}{\partial x} = \frac{\partial w}{\partial t},$$

as well as the equations for the kinetics of the mass transfer between the mobile and bound solutions:

$$\partial(\theta N)/\partial t = -\delta(q, \theta)(N - \gamma C).$$

The coefficient of convective diffusion in a medium with incompletely saturated pores is $D(w, q) = (w - \theta)D_0 + \lambda q$, while D_0 is the coefficient of molecular diffusion and λ is the filtration–dispersion parameter, and the parameters of the exchange kinetics $\gamma \geq 0$, $\delta(q, \theta)$ are dependent on the physicochemical properties of the porous medium and of the solution, all of which are assumed given along with the relationship $p = p(w)$ for the absorptive force of the soil in relation to water content. Equations (1.2) and (1.4) close the above system in the simulation of mass transfer in a swelling porous medium containing colloidal particles.

Consider a semi-infinite layer of soil initially containing an immobile solution of unit concentration, on which is laid a thin layer of fresh water. We assume that the solution penetrates into the soil only under the action of gravity and with complete saturation of the pore space (w = m). We also assume that $\delta(q, \theta) = \alpha q$ and $\gamma = D = 0$ [8]. Then the flushing of the soil is described by the system of equations

$$\theta = \theta^0 + \beta (N - C),$$
$$v = v(\tau) = -\left[1 - \frac{\beta}{m - \theta^0}(N - C)\right]^n \left(\frac{\partial p}{\partial x} - 1\right),$$
$$\frac{\partial}{\partial \tau}(\theta N) = -\alpha v N, \quad -v \frac{\partial C}{\partial x} = \frac{\partial}{\partial \tau}[(m - \theta)C + \theta N]$$

(1.5)

with boundary conditions at the surface of the soil and at the infiltration boundary, respectively:

$$x = 0: \ C = 0, \ p = 0; \ x = s(\tau): \ N = 1, \ p = 0,$$

(1.6)

together with a kinematic condition of the form

$$[m - \theta(s, \tau)]ds/d\tau = v$$

(1.7)

and the initial condition $s(0) = 0$, where $\tau = K_0 t$, $v = q/K_0$, and the functions $C(x, \tau)$ and $N(x, \tau)$ are referred to some characteristic value (here the initial concentration of the soil solution).

2. Approximate Solution. The dimensionless coefficient β appearing in the first equation of (1.5) is dependent on the percent content of clay particles in the soil and does not exceed the value $m - \theta^0 < 1$ when C and N vary in the range $(0, 1)$. We take this coefficient as a small parameter and seek the solution as the asymptotic expansions

$$\theta = \theta_0 + \beta\theta_1 + \beta^2\theta_2 + \ldots, \quad p = p_0 + \beta p_1 + \beta^2 p_2 + \ldots,$$
$$N = N_0 + \beta N_1 + \beta^2 N_2 + \ldots, \quad v = v_0 + \beta v_1 + \beta^2 v_2 + \ldots,$$
$$C = C_0 + \beta C_1 + \beta^2 C_2 + \ldots, \quad s = s_0 + \beta s_1 + \beta^2 s_2 + \ldots$$

(2.1)

Substitution of (2.1) into (1.5), the boundary conditions of (1.6), and the kinematic condition of (1.7) with subsequent equating of terms with identical powers of β leads to a series of linear problems for the functions $\theta_i, N_i, \ldots, s_i$ $(i = 0, 1, 2, \ldots)$.

The system of equations arising in the first stage is

$$\theta_0 = \theta^0, \quad v_0 = -\frac{\partial p_0}{\partial x} + 1, \quad (m - \theta_0)\frac{ds_0}{d\tau} = v_0(\tau),$$
$$\theta_0 \frac{\partial N_0}{\partial \tau} = -\alpha v_0 N_0, \quad -v_0 \frac{\partial C_0}{\partial x} = (m - \theta_0)\frac{\partial C_0}{\partial \tau} + \theta_0 \frac{\partial N_0}{\partial \tau},$$

together with the boundary conditions

$$x = 0: \ C_0 = 0, \ p_0 = 0; \ x = s_0(\tau): \ N_0 = 1, \ p_0 = 0; \ s(0) = 0,$$

and simulates the flushing of a layer of soil without allowance for the clay-particle swelling. The solution is

$$\theta_0 = \theta^0 = \text{const}, \ v_0 \equiv 1, \ p_0 \equiv 0, \ s_0 = \tau/(m - \theta_0),$$
$$N_0 = \exp[\alpha_0(x - s_0)], \ C_0 = \alpha x \exp[\alpha_0(x - s_0)] \ (\alpha_0 = \alpha(m - \theta_0)/\theta_0).$$

(2.2)

We get an uncoupled system of equations for the functions with index 1:

$$\theta_1 = N_0 - C_0, \quad v_1 = -\frac{\partial p_1}{\partial x} - a(N_0 - C_0), \quad (m - \theta_0)\frac{ds_1}{d\tau} = v_1 + \theta_1(s_0, \tau)\frac{ds_0}{d\tau},$$
$$\frac{\partial}{\partial \tau}(\theta_0 N_1 + C_1 N_0) = -\alpha(v_0 N_1 + v_1 N_0),$$
$$-v_0 \frac{\partial C_1}{\partial x} + v_1 \frac{\partial C_0}{\partial x} = \frac{\partial}{\partial \tau}[(m - \theta_0)C_1 + \theta_1 C_0 + \theta_0 N_1 + \theta_1 N_0]$$

(2.3)

with the boundary conditions

$$x = 0: \ C_1 = 0, \ p_1 = 0;$$
$$x = s_0(\tau): \ N_1(s_0, \tau) = -s_1 \frac{\partial N_0}{\partial x}(s_0, \tau), \quad p_1 = 0,$$
$$s_1(0) = 0 \ (a = n/(m - \theta_0)).$$

We get directly from the first equation of (2.3) that

$$\theta_1(x, \tau) = (1 - \alpha x)\exp[\alpha_0(x - s_0)].$$

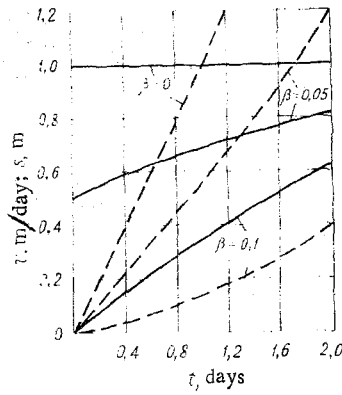Integration of the second equation on the basis of the boundary condition $p_1(0, \tau) = 0$ leads to

Fig. 1

$$p_1(x, \tau) = - v_1 x - a\alpha_0^{-1} e^{-z_0} \{e^\xi - 1 + \alpha\alpha_0^{-1} [(1 - \xi) e^\xi - 1]\}$$
$$(z_0 = \alpha_0 s_0(\tau), \quad \xi = \alpha_0 x).$$

The function v_1 is derived from the condition $p_1(s_0, \tau) = 0$ and takes the form

$$v_1(z_0) = \frac{am}{m - \theta_0} \left[\frac{\theta_0}{m} - \frac{1 - \exp(-z_0)}{z_0} \right].$$

We introduce the function $z_1 = \alpha_0(m - \theta_0)s_1(\tau)$ and substitute the value of $\theta_1(s_0, \tau)$ from the third equation of (2.3) and the initial condition $z_1(0) = 0$ to get

$$z_1(z_0) = \int_0^{z_0} \left[\frac{\alpha_0 - \alpha z}{\alpha_0 (m - \theta_0)} + v_1(z) \right] dz.$$

Substitution of these relationships transforms the fourth equation in (2.3) and a corresponding boundary condition to

$$\frac{\partial N_1}{\partial z_0} + N_1 = e^{-z_0}\varphi(\xi, z_0), \quad \xi = z_0: \quad N_1 = - z_1/(m - \theta_0)$$
$$\left(\varphi(\xi, z) = \frac{2}{\theta_0} \left(1 - \frac{\alpha}{\alpha_0} \xi \right) \exp(2\xi - z) - v_1(z) \exp \xi \right).$$

The solution to this is the function

$$N_1 = e^{-z_0}\left[\int_\xi^{z_0} \varphi(\xi, z)\, dz - e^\xi z_1(\xi)(m - \theta_0)^{-1} \right].$$

Similarly, to determine $C_1(\xi, z_0)$ we have the boundary-value problem

$$\frac{\partial C_1}{\partial \xi} + \frac{\partial C_1}{\partial z_0} = \frac{\alpha}{\alpha_0} \varphi_1(\xi, z_0),$$
$$\xi = 0: \quad C_1 = 0$$
$$\left(\varphi_1(\xi, z) = N_1(\xi, z) - \frac{2\alpha}{\theta_0 \alpha_0} \xi \left(1 - \frac{\alpha}{\alpha_0} \xi \right) e^{2(\xi - z)} - \xi v_1(z) e^{\xi - z} \right).$$

The solution is written as

$$C_1(\xi, z_0) = \frac{\alpha}{\alpha_0} \int_0^\xi \varphi_1(u, u + z_0 - \xi)\, du.$$

The latter terms in the expansions of (2.1) can be derived analogously, although there are certain difficulties of computational character.

The solid and broken lines in Fig. 1 show correspondingly the graphs for $v(\tau)$ (infiltration rate) and $s(\tau)$ (washing front) as calculated for various values of β using the first two terms in the asymptotic expansions of (2.1). The following initial values were used: $K_0 = 0.2$ m/day, $\theta^0 = 0.2$, $m = 0.4$, $\alpha = 0.4$ m^{-1}, $a = 10$; the

calculations show that the swelling substantially influences the infiltration into the clay soil.

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GROWTH AND COLLAPSE OF VAPOR BUBBLES IN
BOILING LIQUID

F. B. Nagiev and N. S. Khabeev

A study is made of the dynamics and heat-mass exchange of vapor bubbles in water and cryogenic liquids under the action of an abrupt pressure change, which corresponds to bubble behavior in a shock wave, when the wave enters a bubble curtain. Behavior under varying pressure is also studied.

A system of basic equations describing the heat-mass exchange processes and dynamics of a spherical homobaric bubble in a liquid was presented in [1]. The equations of heat adflux, continuity, and state in spherical Euler coordinates (r, t) have the form

$$\rho_v\left(\frac{\partial u_v}{\partial t} + v_v\,\frac{\partial u_v}{\partial r}\right) = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(\lambda_v r^2\,\frac{\partial T_v}{\partial r}\right) + \frac{p_v}{\rho_v}\,\frac{d\rho_v}{dt},$$

$$\frac{\partial \rho_v}{\partial t} + \frac{1}{r^2}\,\frac{\partial}{\partial r}\,(r^2\rho_v v_v) = 0, \quad p_v(t) = B\rho_v(r,t)\,T_v(r,t),$$

$$\rho_l\left(\frac{\partial u_l}{\partial t} + v_l\,\frac{\partial u_l}{\partial r}\right) = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(\lambda_l r^2\,\frac{\partial T_l}{\partial r}\right),$$

$$v_l = w_l R^2/r^2, \quad u_l = c_l T_l, \quad u_v = c_V T_v, \quad \rho_l = \text{const},$$

(1)

where ρ is the density; T, temperature; p, pressure; v, velocity; u, specific internal energy; λ, thermal conductivity coefficient; R, bubble radius; w_l, mass velocity of liquid on bubble surface; B, gas constant; c_V, specific heat of vapor at constant volume. The subscripts l and v refer to liquid and vapor parameters, respectively, while the subscript 0 indicates parameters in the unperturbed state.

The boundary conditions for the heat adflux equations have the form

$$r = 0, \ \partial T_v/\partial r = 0,$$

$$r = \infty, \ T_l = T_0,$$

$$r = R(t), \quad \lambda_l\,\frac{\partial T_l}{\partial r} - \lambda_v\,\frac{\partial T_v}{\partial r} = jl, \quad T_v = T_l = T_s(p_v),$$

(2)

where $T_s(p_v)$ is the saturation temperature; j is the rate of phase conversion per unit surface; l is the latent heat of evaporation. The last condition defines the so-called quasi-equilibrium approximation. The bubble

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